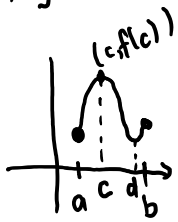
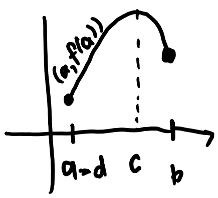
Lecture 13

Tuesday, October 4, 2016 8:13 AM

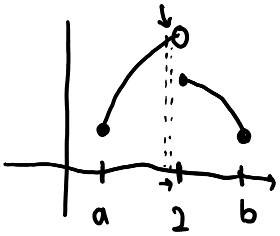
THE EXTREME VALUE THEOREM

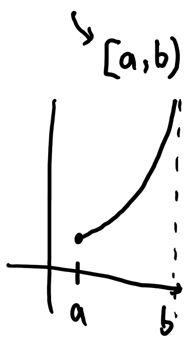
If f is <u>continuous</u> on a <u>closed interval</u> $[a,b]$, then f attains an abs max value $f(c)$ and an abs min $f(d)$, for some c,d in $[a,b]$.

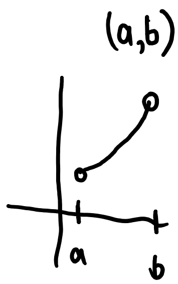




Rmk Continuity and closed intervals are important.

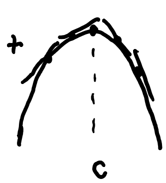






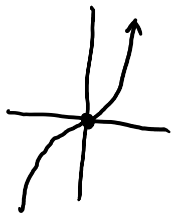
FERMAT'S THEOREM

If f has a local max or local min at c, and if $f'(c)$ exists, then $f'(c) = 0$

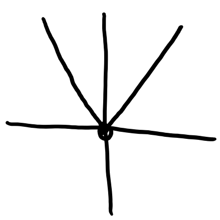


Rmk 1 The converse is not true.

$f(x) = x^3$, $f'(x) = 3x^2$, $f'(0) = 0$



a) There may be local max or min where $f'(c)$ DNE.



$f(x) = |x|$ has local (& abs) min @ 0, but $f'(0)$ DNE.

DEF A critical number of a func f is a number c in the domain of f, where $f'(c) = 0$ or $f'(c)$ DNE.

Ex Find C.N. of $f(x) = x^{3/5}(4-x)$

$$= 4x^{3/5} - x^{8/5}$$

$$f'(x) = 4 \cdot \frac{3}{5} x^{-2/5} - \frac{8}{5} x^{3/5}$$

$$= \frac{12}{5} x^{-2/5} - \frac{8}{5} x^{3/5}$$

$$= \frac{4}{5} x^{-2/5} \left[3 - 2x \right]$$

$$= \frac{4}{5}x^{-2/5}\left[3 - 2x\right]$$

$$f'(x) = \frac{4(3-2x)}{5x^{2/5}}$$

<u>Then</u> $f'(x) = 0$ if $4(3-2x) = 0 \Rightarrow x = \frac{3}{2}$

<u>Also</u> $f'(x)$ DNE if $5x^{2/5} = 0 \Rightarrow x = 0$.

CN are $x = 0, \frac{3}{2}$.

<u>Restating Fermat's Thm</u>

If f has a local max/min @ c, then c is a critical number of f.

<u>CLOSED INTERVAL METHOD</u> :

To find abs max & min of a cont func f on a closed interval $[a, b]$:

1) Find all CN of f in (a, b).

2) Find the values of f at the C.N and the endpoints.

3) The largest and smallest values from

Step 2 are the abs max and abs min resp.

Ex Find abs max and abs min of

$f(x) = x^3 - 3x + 1$ on the interval $[0,3]$?

- Since f is a polynomial it is continuous on the closed interval $[0,3]$, we can use the closed interval method.

STEP 1 $f(x) = x^3 - 3x + 1$

$f'(x) = 3x^2 - 3$

$f'(x) = 0 \Rightarrow 3x^2 - 3 = 0 \Rightarrow$

$3x^2 = 3 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$.

-1 is not in $(0,3)$, so we can disregard it.

$f(x) = x^3 - 3x + 1$

STEP 2 endpts

$f(0) = 0^3 - 3.0 + 1 = 1$

$f(3) = 3^3 - 3.3 + 1 = 19$

C.N. $f(1) = 1^3 - 3.1 + 1 = -1$

STEP 3 Abs max is $f(3) = 19$

STEP 3 Abs max is $f(3) = 19$

Abs min is $f(1) = -1$

4.2 THE MEAN VALUE THEOREM.

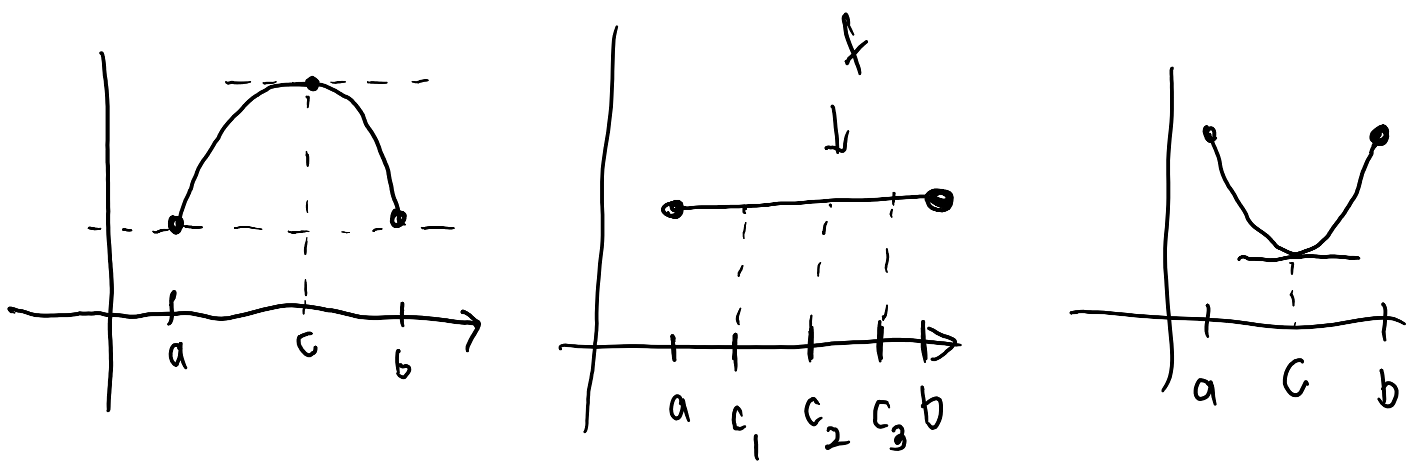
ROLLE'S THEOREM

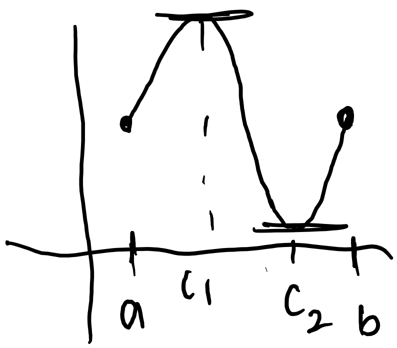
Let f be a function satisfying the following 3 hypothesis:

1) f is continuous on $[a,b]$.
2) f is differentiable on (a,b).
3) $f(a) = f(b)$

Then there is a number c in (a,b) such that $f'(c) = 0$.

$f(x) = d$





<u>Ex</u> Prove that $f(x) = x^3 + x - 1$ has <u>exactly one root</u>.

<u>STEP 1</u> Show it has atleast 1 root.

<u>Use IVT</u> $f(0) = 0^3 + 0 - 1 = -1 < 0$

$f(1) = 1^3 + 1 - 1 = 1 > 0$

Since f is a polynomial, it is continuous, and by IVT, there is a number c betn 0 & 1 s.t $f(c) = 0$.

Thus f has a root.

<u>STEP 2</u> We would like to show that there cannot be more than 1 root.

Use Rolle's Thm and <u>argue by contradiction</u>.